



1. Skupovi brojeva

9.10.2020.

Slavni skupovi brojeva

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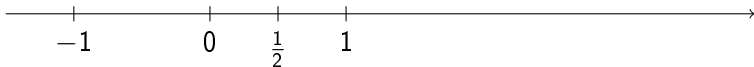


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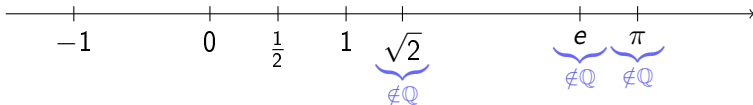


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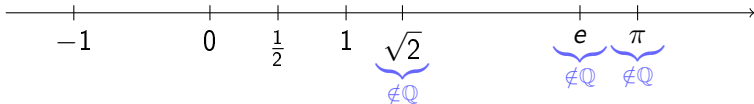
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- $\mathbb{C} = \{x + iy : x, y \in \mathbb{R}\}$.

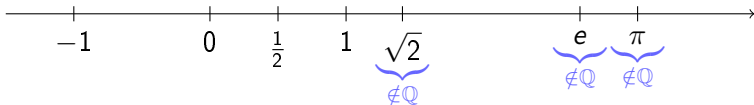
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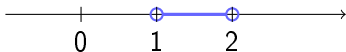
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Neka su $a, b \in \mathbb{R}$.

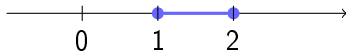
- **Otvoreni interval:** $\langle a, b \rangle := \{x \in \mathbb{R} : a < x < b\}$.

PR.: $\langle 1, 2 \rangle$



- **Zatvoreni interval ili segment:** $[a, b] := \{x \in \mathbb{R} : a \leq x \leq b\}$.

PR.: $[1, 2]$

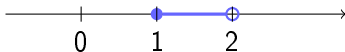


- **Poluotvoreni intervali:** $\langle a, b] i [a, b \rangle$.

PR.: $\langle 1, 2]$



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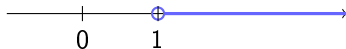
Napomena.

- Simbol “:=” se čita “definira se kao”.

- **Otvoreni polupravci:**

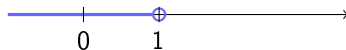
- $\langle a, +\infty \rangle := \{x \in \mathbb{R} : x > a\}$

PR.: $\langle 1, +\infty \rangle$



- $\langle -\infty, a \rangle := \{x \in \mathbb{R} : x < a\}$

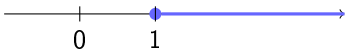
PR.: $\langle -\infty, 1 \rangle$



- **Zatvoreni polupravci:**

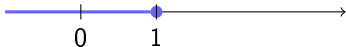
- $[a, +\infty) := \{x \in \mathbb{R} : x \geq a\}$

PR.: $[1, +\infty)$

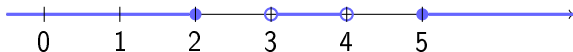


- $\langle -\infty, a] := \{x \in \mathbb{R} : x \leq a\}$

PR.: $\langle -\infty, 1] \rangle$



$$\text{PR.: } \langle -\infty, 2 \rangle \cup \langle 3, 4 \rangle \cup [5, +\infty)$$



Općenito, **unija skupova** A i B je skup

$$\begin{aligned} A \cup B &:= \{x : x \text{ je element barem jednog od skupova } A \text{ i } B\} \\ &= \{x : x \in A \text{ ili } x \in B\}. \end{aligned}$$

Napomena. U matematici je “ili” uvijek inkluzivno ili, tj.

ili := barem jedno od.

Primjer 1

Skicirajmo skup

$$S := \langle 1, 2] \cup [2, 3]$$

na brojevnom pravcu.

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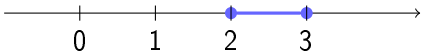
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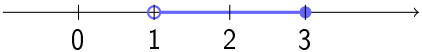
$\langle 1, 2 \rangle$



$[2, 3]$



S

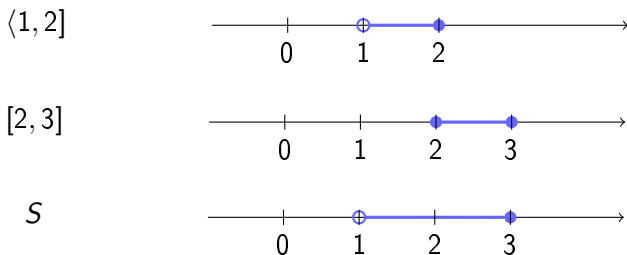


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Dakle,

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Primjer 2

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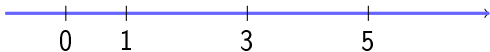
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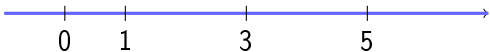
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S



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Presjek skupova A i B je

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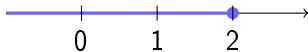
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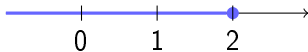


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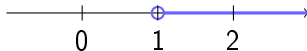
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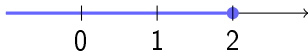


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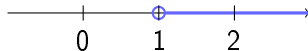
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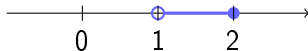
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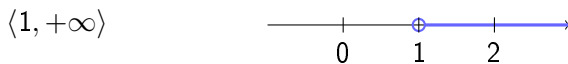
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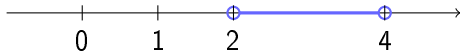
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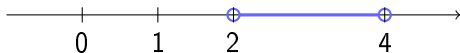
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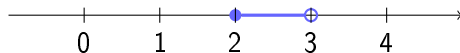
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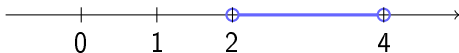
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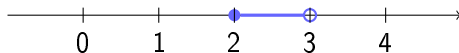
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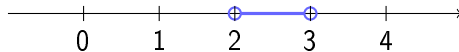
$$\langle 2, 4 \rangle$$



$$[2, 3)$$



$$\underbrace{\langle 2, 4 \rangle \cap [2, 3)}_{\langle 2, 3 \rangle}$$



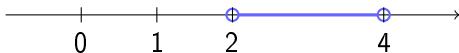
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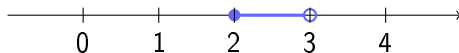
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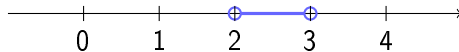
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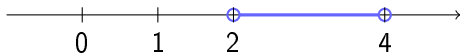
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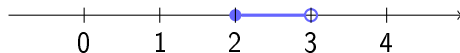
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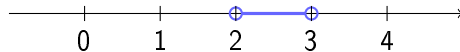
$\langle 2, 4 \rangle$



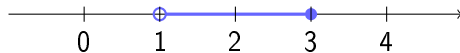
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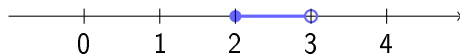
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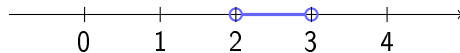
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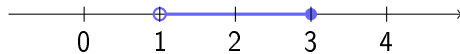
$[2, 3]$



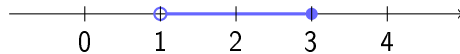
$\underbrace{\langle 2, 4 \rangle \cap [2, 3]}_{\langle 2, 3 \rangle}$



$\langle 1, 3 \rangle$



S



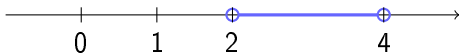
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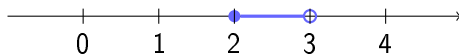
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na brojevnom pravcu.

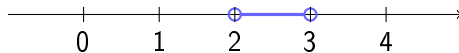
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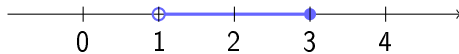
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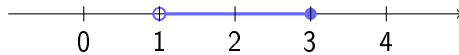
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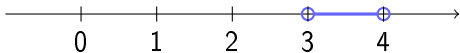
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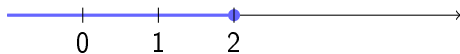
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Skicirajmo skup

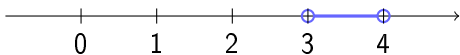
$$S := \langle -\infty, 2] \cap \langle 3, 4 \rangle$$

na brojevnom pravcu.

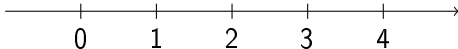
$\langle -\infty, 2]$



$\langle 3, 4 \rangle$



S



Primjer 4

Skicirajmo skup

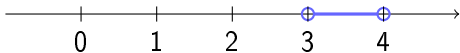
$$S := \langle -\infty, 2] \cap \langle 3, 4 \rangle$$

na brojevnom pravcu.

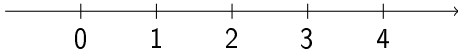
$\langle -\infty, 2]$



$\langle 3, 4 \rangle$



S



Dakle,

$S = \emptyset :=$ skup bez ijednog elementa, **prazan skup**.

Razlika skupova A i B je skup

$$A \setminus B := \{x : x \in A, \text{ ali } x \notin B\}.$$

Napomena.

- Simbol “ \notin ” se čita “nije element iz”, a simbol “ \setminus ” se ponekad čita “bez”.

Razlika skupova

Razlika skupova A i B je skup

$$A \setminus B := \{x : x \in A, \text{ ali } x \notin B\}.$$

PR.: Skicirajmo skup

$$S := \mathbb{R} \setminus \{0\}$$

na brojevnom pravcu.

Napomena.

- Simbol “ \notin ” se čita “nije element iz”, a simbol “ \setminus ” se ponekad čita “bez”.

Razlika skupova

Razlika skupova A i B je skup

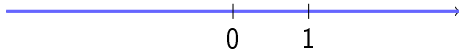
$$A \setminus B := \{x : x \in A, \text{ ali } x \notin B\}.$$

PR.: Skicirajmo skup

na brojevnom pravcu.

$$S := \mathbb{R} \setminus \{0\}$$

\mathbb{R}



Napomena.

- Simbol “ \notin ” se čita “nije element iz”, a simbol “ \setminus ” se ponekad čita “bez”.

Razlika skupova

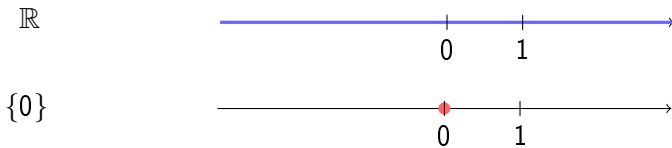
Razlika skupova A i B je skup

$$A \setminus B := \{x : x \in A, \text{ ali } x \notin B\}.$$

PR.: Skicirajmo skup

na brojevnom pravcu.

$$S := \mathbb{R} \setminus \{0\}$$



Napomena.

- Simbol “ \notin ” se čita “nije element iz”, a simbol “ \setminus ” se ponekad čita “bez”.

Razlika skupova

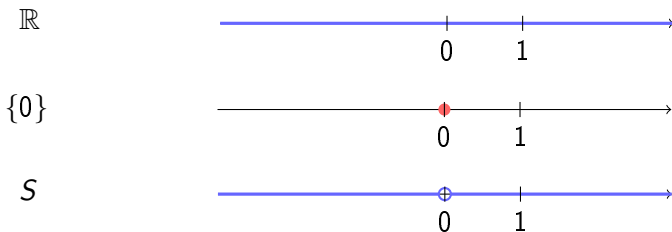
Razlika skupova A i B je skup

$$A \setminus B := \{x : x \in A, \text{ ali } x \notin B\}.$$

PR.: Skicirajmo skup

na brojevnom pravcu.

$$S := \mathbb{R} \setminus \{0\}$$



Napomena.

- Simbol “ \notin ” se čita “nije element iz”, a simbol “ \setminus ” se ponekad čita “bez”.

Razlika skupova

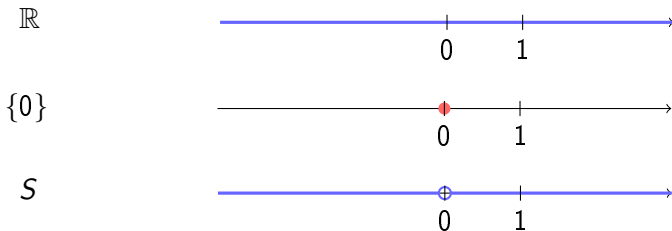
Razlika skupova A i B je skup

$$A \setminus B := \{x : x \in A, \text{ ali } x \notin B\}.$$

PR.: Skicirajmo skup

na brojevnom pravcu.

$$S := \mathbb{R} \setminus \{0\}$$



Primijetimo da je

$$S = \langle -\infty, 0 \rangle \cup \langle 0, +\infty \rangle.$$

Napomena.

- Simbol “ \notin ” se čita “nije element iz”, a simbol “ \setminus ” se ponekad čita “bez”.

Primjer 5

Skicirajmo skup

$$S := \langle 1, +\infty \rangle \setminus \{0, 1, 2\}$$

na brojevnom pravcu.

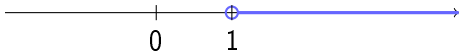
Primjer 5

Skicirajmo skup

$$S := \langle 1, +\infty \rangle \setminus \{0, 1, 2\}$$

na brojevnom pravcu.

$$\langle 1, +\infty \rangle$$



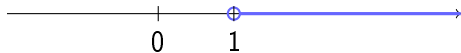
Primjer 5

Skicirajmo skup

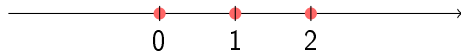
$$S := \langle 1, +\infty \rangle \setminus \{0, 1, 2\}$$

na brojevnom pravcu.

$\langle 1, +\infty \rangle$



$\{0, 1, 2\}$



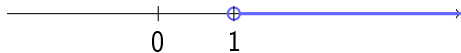
Primjer 5

Skicirajmo skup

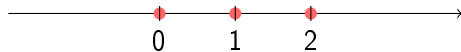
$$S := \langle 1, +\infty \rangle \setminus \{0, 1, 2\}$$

na brojevnom pravcu.

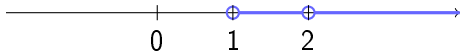
$\langle 1, +\infty \rangle$



$\{0, 1, 2\}$



S

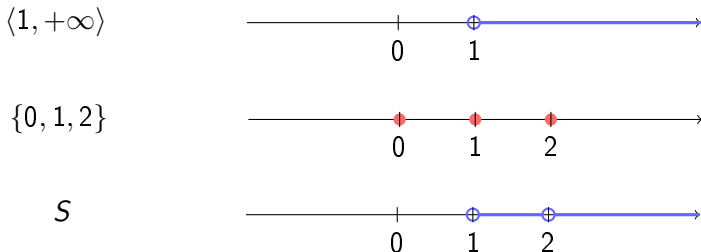


Primjer 5

Skicirajmo skup

$$S := \langle 1, +\infty \rangle \setminus \{0, 1, 2\}$$

na brojevnom pravcu.



Dakle,

$$S = \langle 1, 2 \rangle \cup \langle 2, +\infty \rangle = \langle 1, +\infty \rangle \setminus \{2\}.$$

Primjer 6

Skicirajmo skup

$$S := [0, 2) \setminus \left[\frac{1}{2}, 1\right]$$

na brojevnom pravcu.

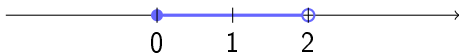
Primjer 6

Skicirajmo skup

$$S := [0, 2) \setminus \left[\frac{1}{2}, 1\right]$$

na brojevnom pravcu.

$[0, 2)$



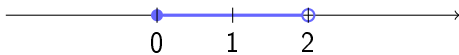
Primjer 6

Skicirajmo skup

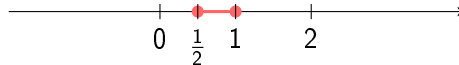
$$S := [0, 2) \setminus [\frac{1}{2}, 1]$$

na brojevnom pravcu.

$[0, 2)$



$[\frac{1}{2}, 1]$



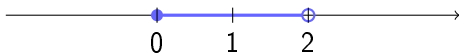
Primjer 6

Skicirajmo skup

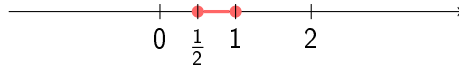
$$S := [0, 2) \setminus [\frac{1}{2}, 1]$$

na brojevnom pravcu.

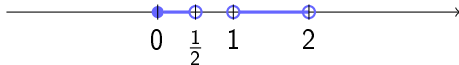
$[0, 2)$



$[\frac{1}{2}, 1]$



S



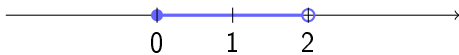
Primjer 6

Skicirajmo skup

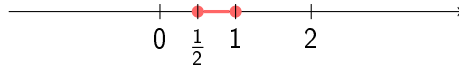
$$S := [0, 2) \setminus [\frac{1}{2}, 1]$$

na brojevnom pravcu.

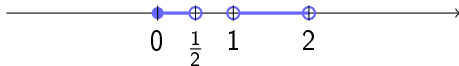
$[0, 2)$



$[\frac{1}{2}, 1]$



S



Dakle,

$$S = [0, \frac{1}{2}) \cup (1, 2).$$

Primjer 7

Skicirajmo skup

$$S := [0, 4] \setminus (\langle 1, 2 \rangle \cup \langle 3, 4 \rangle)$$

na brojevnom pravcu.

Primjer 7

Skicirajmo skup

$$S := [0, 4] \setminus (\langle 1, 2 \rangle \cup \langle 3, 4 \rangle)$$

na brojevnom pravcu.

$[0, 4]$



Primjer 7

Skicirajmo skup

$$S := [0, 4] \setminus (\langle 1, 2 \rangle \cup \langle 3, 4 \rangle)$$

na brojevnom pravcu.

$[0, 4]$



$\langle 1, 2 \rangle \cup \langle 3, 4 \rangle$



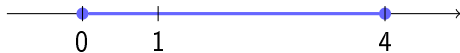
Primjer 7

Skicirajmo skup

$$S := [0, 4] \setminus (\langle 1, 2 \rangle \cup \langle 3, 4 \rangle)$$

na brojevnom pravcu.

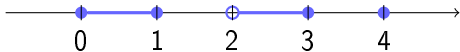
$[0, 4]$



$\langle 1, 2 \rangle \cup \langle 3, 4 \rangle$



S



Primjer 7

Skicirajmo skup

$$S := [0, 4] \setminus (\langle 1, 2 \rangle \cup \langle 3, 4 \rangle)$$

na brojevnom pravcu.

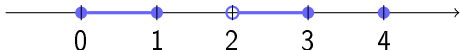
$[0, 4]$



$\langle 1, 2 \rangle \cup \langle 3, 4 \rangle$



S



Dakle,

$$S = [0, 1] \cup \langle 2, 3 \rangle \cup \{4\} .$$